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A.D. Patel Institute of Technology-New V.V.Nagar

Mid semester Examination, March 2009

B.E. Sem II (All Branch)

Subject:-Mathematics II

Time:- 8.00 am to 9.00am **Total marks :-20**

Instruction:

1. Figures to the right indicate full marks.

2. Attempt all questions.

Q.1) **5**

(a) **Do as Directed**

(i) Which of the following are linear equations in x_1, x_2, x_3 ?

$a) x_1 + 5x_2 - \sqrt{2}x_3 = 1$ $b) x_1 + 3x_2 + x_1x_3 = 2$ $c)\ x_1 = -7x_2 + 3x_3$

$d) x_1^{-2} + x_2 + 8x_3 = 5$ $d)\ x_1^{\frac{3}{5}} - 2x_2 + x_3 = 4$ $e)\ \pi x_1 - \sqrt{2}x_2 + \frac{1}{3}x_3 = 7^{\frac{1}{3}}$

(ii) For which value(s) of the constant K does the system

$$x - y = 3$$
$$2x - 2y = k$$

Have infinitely many solutions?

(iii) If **u** and **v** are orthogonal vectors in $\mathbf{R}^n$ such that $\|\mathbf{u}\| = 1$ and $\|\mathbf{v}\| = 1$, then $d(\mathbf{u}, \mathbf{v})$?

(iv) (-2, 0, 1), (3, 2, 5), (6,-1, 1), (7, 0,-2) sets of vectors in $\mathbf{R}^4$ are linearly dependent?

(v) If A is 3x5, matrix then the rank of A is at most?

(b) Attempt **Any ONE** **3**

(i) Solve the following systems by Gauss- Jordan Method

$$3x_1 + 2x_2 - x_3 = -15$$
$$5x_1 + 3x_2 + 2x_3 = 0$$
$$3x_1 + x_2 + 3x_3 = 11$$
$$-6x_1 - 4x_2 + 2x_3 = 30$$

(ii) Obtain solution of a linear system using Inverse of a matrix method

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 + 8x_3 = 17$$

(c)

(i) Obtain Norm and Distance , If $\mathbf{u} = (1,3,-2,7)$ and $\mathbf{v} = (0,7,2,2)$ in the Euclidean space $\mathbf{R}^4$ **1**

(ii) Verify that the Cauchy-Schwarz inequality holds **1**

$\mathbf{u} = (0,-2,2,1)$, $\mathbf{v} = (-1,-1,1,1)$

(iii) For which values of k are $\mathbf{u}$ and $\mathbf{v}$ Orthogonal? **1**

$\mathbf{u} = (k,k,1)$, $\mathbf{v} = (k,5,6)$

Q. 2)

(a) Attempt **Any ONE** **3**

(i) Let V be the set of positive real numbers with addition and scalar multiplication defined as follows

$x + y = xy$ and $kx = x^k$

Show that V is a Vector Space.

(ii) The set of 2x2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with the standard matrix addition and scalar multiplication. Determine for those that are not vector space, list all axioms that fail to hold.

(b) Attempt **Any ONE** **3**

(i) Find the coordinate vector of w relative to the basis $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ of $\mathbf{R}^3$.

$\mathbf{w} = (5,-12,3), \mathbf{u}_1 = (1,2,3), \mathbf{u}_2 = (-4,5,6), \mathbf{u}_3 = (7,-8,9)$

(ii) Let $\mathbf{u}_1 = (1,2,1), \mathbf{u}_2 = (2,9,0)$ and $\mathbf{u}_3 = (3,3,4)$. Show that the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a basis for $\mathbf{R}^3$.

(c) Attempt **Any ONE** **3**

(i) For the matrix A find basis for Column space, Null space of A, Also find rank(A) and nullity(A)

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

(ii) Find a basis for the subspace of $\mathbf{R}^4$ spanned by the given vectors.

(1,1,-4,-3),(2,0,2,-2),(2,-1,3,2)